Spatial Sparse Constraint in the Transmembrane Potential Based ECG Inverse Problem

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Abstract

The aim of Electrocardiographic (ECG) inverse problem is to use the measured ECG signals on the body surface to noninvasively reconstruct the activity of heart. Due to the ill-posedness, the solution of ECG inverse problem needs employ as much prior information about the cardiac activities as possible. In this study, the spatial sparse performance of the transmembrane potential (TMP) was investigated as a priori constraint to tackle the TMP based ECG inverse problem for the first time. The existence of the spatial sparseness was detailed analyzed and a novel spatial variation operator in terms of the spatial connection relationship of the heart mesh was proposed to describe it, and combined as a L1 norm penalty term into the solution of the TMP based ECG inverse problem. The iteratively reweighted norm (IRN) algorithm was used to solve the L1 norm based problem. With the simulation study based on the virtual heart and realistic volume conductor model, the proposed method was compared to the common Tikhonov method with zero order and spatial Laplacian operators to reconstruct the TMP on the cardiac surface. The results demonstrated that the spatial sparseness constraint is successfully combined into the ECG inverse problem and more accurate TMP distribution can be obtained.

1. Introduction

Electrocardiographic (ECG) inverse problem is a noninvasive and quantitative method to detect the electrophysiological activity of heart from the measured body surface potentials (BSPs). More functional information about the heart can be obtained from the ECG inverse problem compared to the standard 12-lead ECG, which is commonly used in clinical diagnosis. Unfortunately, the ECG inverse problem is an ill-posed problem, which needs the regularization process. The nature of regularization is to introduce as much prior information as possible, which relates to the "source", i.e. the cardiac activity.

In this study, new prior information about spatial sparseness, associating with transmembrane potential (TMP) distribution, was explored in the TMB based ECG inverse problem.

To include the constraint of TMP, the ECG inverse problem with equivalent double layer (EDL) model was used [1]. By solving the govern equation of EDL based ECG problem using boundary element method, the relationship between TMP Φ_M and BSP Φ_T can be expressed as [2]

$$T_{BM}\Phi_M = \Phi_B \tag{1}$$

where T_{BM} is the transfer matrix (or lead field matrix) associated with the volume conductor model properties including geometry and conductivities.

Accordingly, the regularization process of ECG inverse problem is expressed as a constraint optimization problem[3]

$$\min\left\{\frac{1}{2}\left\|T_{BM}\Phi_M - \Phi_B\right\|_2^2 + \lambda C(\Phi_M)\right\} \tag{2}$$

where $C(\Phi_M)$ is the constraint term, which includes the prior information about source TMP. λ (>0) is the regularization parameter, which controls the trade-off between the two terms: data fitting and constraint terms. The difference between various methods in ECG inverse problem mainly lies in the design of the constraint term, and many spatio-temporal constraints related to the cardiac electrophysiological activities have been developed (refer to [4] and the references cited therein).

In present study, we explored a novel spatial constraint, which related to the sparse performance of TMP during the ventricular depolarization. A variation operator with L1 norm was proposed to describe this constraint and combined into Eq. (2). Using simulated data from realistic heart-torso model, the proposed method was compared to commonly used Tikhonov methods with zero order and spatial Laplacian operators [3,5].

2. Theory and method

2.1. Spatial sparseness in TMP distribution

Cardiac TMP (or membrane potential) is electrical potential difference between the interior and exterior of a myocye, which relates to action potential (AP) waveform of specific myocyte and activation sequence. Fig.1 displayed a typical AP waveform, which was simulated by ten Tusscher model of human ventricular myocyte [6]. The AP includes five phases: resting potential (phase 4), fast depolarization (phase 0), intial repolarization (phase 1), plateau (phase 2) and rapid repolarization (phase 3). Before the activation, the myocyte is inactive and the TMP is the resting potential (about -90 mV). Once the myocyte become active, the TMP rapidly increases to the highest potential (about +30mV). Based on this property, on the whole heart level, there is a large variation of TMP (from the resting potential to the overshoot potential) on the cells, which locates around the boundaries between active and inactive cardiac regions at each time instant during ventricular depolarization. Such performance can be seen as the spatial sparseness of TMP, which is potentially useful prior information of TMP and introduced to tackle the TMP based ECG inverse problem in present study.

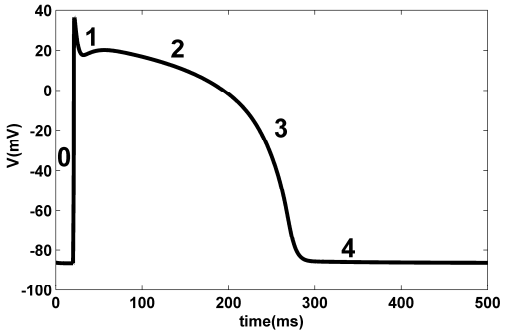


Figure 1. A typical AP waveform simulated by ten Tusscher model of human ventricular myocyte.

2.2. A spatial variation operator

In order to describe the spatial sparseness of TMP mathematically, a novel spatial variation operator V in terms of the cardiac mesh was proposed as follows

$$V = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1N} \\ v_{21} & v_{22} & \cdots & v_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ v_{M1} & v_{M2} & \cdots & v_{MN} \end{bmatrix} \tag{3}$$

$$\begin{cases} v_{ij} = l; \quad v_{ik} = -l \ \ if\ element\ j,k\ share\ edge\ i \\ v_{ij} = 0; \qquad\qquad otherwise \end{cases}$$

where $\boldsymbol{V}$ is a M × N matrix, M and N are the total number of edges and nodes of cardiac triangular elements, and l is the length of edge i. The triangular mesh guarantees that each edge is only shared by two elements; therefore, each row of $\boldsymbol{V}$ only has two nonzero values, i.e. l and $-l$, which correspond to the two elements with same edge i. The introduction of length of edge performed as kind of weight.

To obtain the sparse solution of TMP, the penalty term with operator V was formulated into L1 norm, and substituted into Eq. (1) as

$$\min\left\{\frac{1}{2}\left\|\mathrm{T}_{BM}\Phi_{\mathrm{M}} - \Phi_{\mathrm{B}}\right\|_{L2}^{2} + \lambda\left\|V\Phi_{M}\right\|_{L1}\right\} \tag{4}$$

2.3. Solver for L1 norm regularization

The non-differentiability of L1-norm term made the solution of the objective function more complicated. Fortunately, a multitude of algorithms have been developed to solve the L1-norm-based problem [7]. In this study, we chose the iteratively reweighted norm (IRN) algorithm to handle the problem. The non-differentiable term was replaced as a quadratic term by introducing a small positive value. The IRN algorithm is summarized as follows.

IRN algorithm

Initialization

$$\Phi_M^{(0)} = (T_{BM}^T T_{BM} + \lambda V^T V)^{-1} T_{BM}\Phi_B$$

For step k=0,1,...

$$W_{\Phi_M}^{(k)} = diag\left(\left(\left|V\Phi_M^{(k)}\right| + \beta\right)^{-1}\right)$$

$$\Phi_M^{(k)} = (T_{BM}^T T_{BM} + \lambda T_{BM}^T W_{\Phi_M}^{(k)} T_{BM})^{-1} T_{BM}^T \Phi_B$$

Stopping criterion

If $\left[\frac{f(\Phi_M^{(k+1)})}{f(\Phi_M^{(k)})} - 1\right] \le \varepsilon$, (ε is a specified tolerance), then STOP;

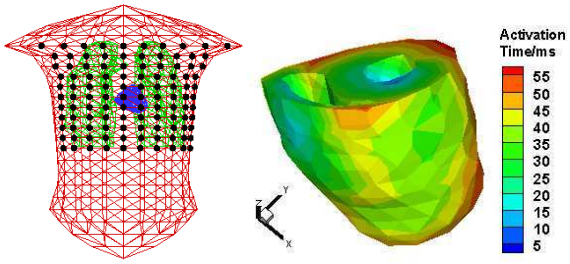
else k=k+1and go to step k.

In the above algorithm, ε , the specified tolerance, was set to be 0.01, and β is a small positive constant with an assigned value of 10^{-5} in this work.

2.4. Simulation protocol

The proposed method was investigated into a model-based simulation study, in which a realistic heart-lung-torso model was taken. The TMP on the ventricular nodes, which located on the endo- and epicardial surfaces, was calculated after the excitation conduction simulation on the ventricle. For the more details about this model, please refer to [8]. Fig.2 (A) displayed the meshes of ventricle, lung and torso surfaces. And Fig.2 (B) illustrated the EDL model with activation time sequence, which included the endo-and epi-cardial ventricular

surfaces illustrated. There were 297 nodes and 586 elements for lung, 412 nodes and 820 elements for torso. The BSPs with 110 leads (shown as the black nodes) were used for inverse calculation to simulate the realistic measurement with limited leads. In terms of EDL model, we didn't consider the surface with all nodes; instead, a down-sample model was used, which had 316 nodes and 628 elements. The measured BSPs were simulated by calculation from the TMP-based ECG forward problem with added Gaussian noise at different signal-to-noise ratio (SNR) (e.g. 30dB and 50dB). The proposed constraint matrix with L1 (L1_V) and L2 norm (L2_V) were compared to the common Tikhonov methods with zero order (L2_I) and Laplacian operators (L2_L) [5]. Since the TMP was restricted within lower and upper bounds (*a priori*), in this simulation study, which were set to -90 and 0 mV, respectively. Therefore, it become a minimization problem with inequality constraints, which was solved by *MOSEK* toolbox [9] both in the L1 and L2 norm problems to get TMP solutions, e.g. $\Phi_M^{(k)}$ in IRN algorithm.



(A) (B)

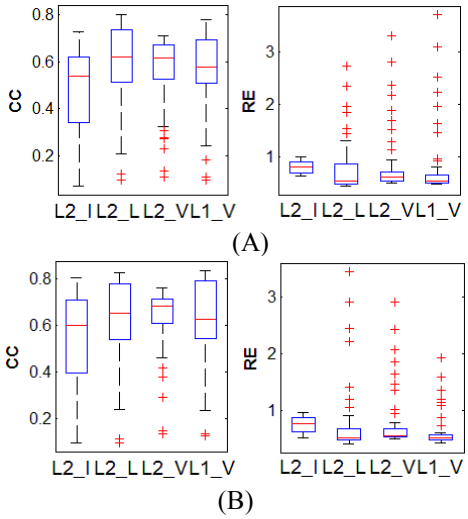
Figure 2. The simulation model: (A) the meshes of ventricle, lung and torso surfaces, (B) the EDL model with activation time sequence.

Correlation Coefficient (CC) and Relative Error (RE) were used to quantitatively evaluate the accuracy of the reconstructed TMPs.

3. Results

The choice of optimal regularization parameter was not the focus of this study, therefore, the regularization parameter was chosen as the optimal one with highest CC by comparing 30 different regularization parameters' solutions.

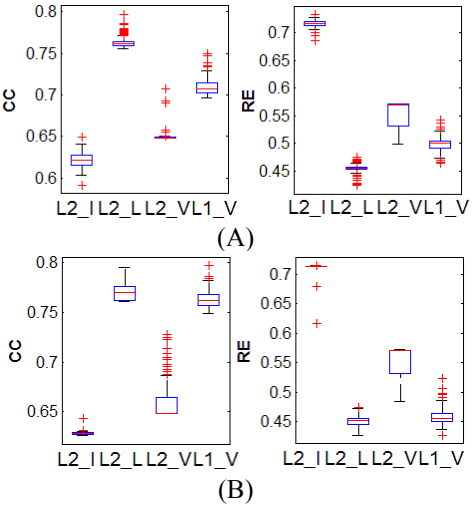
In first simulation study, we compared the performance of reconstructed TMPs from 1~55 ms' BSPs using four different methods. Fig.3 showed the corresponding CC and RE with boxplot. It can be seen that L2_I had the worst results, though no outliers existed in it compared to that of other three methods. In fact, the RE and CC all fluctuated a lot for different time's results in all methods, which may be since no temporal constraints were taken as that in [5]. In terms of CC and RE, L2_V achieved a little bit better solutions than L2_L and L1_V, while L1_V didn't get the expected best results.



(A)

(B)

Figure 3. Boxplot of CC and RE for reconstructed TMPs from 1~55ms' BSPs by four methods: (A) SNR: 30dB; (B) SNR: 50dB.



(A)

(B)

Figure 4. Boxplot of CC and RE for reconstructed TMPs from Monte Carlo simulation of 30 ms' BSP by four methods: (A) SNR: 30dB; (B) SNR: 50dB.

We further compared the four different methods by Monte Carlo simulation of 100 times using only the 30 ms' BSP. The CC and RE with boxplot was shown in Fig.4 with (A) for 30dB and (B) for 50 dB. From Fig.4, it can be seen that L2_L had the best performance in terms of RE and CC. However, when we checked the reconstructed TMP distribution, some interesting phenomenon can be found. One example of reconstructed TMP distribution from BSPs with 30dB SNR by four different methods was shown in Fig.5. It can be seen that L1_V obtained the sharp TMP distribution and the boundary between active and inactive areas (seen in Fig.5 (E)) was much clear in solution of L1_V, though largest CC was found in L2_L. More artifacts were existed in L2_L than that of L1_V.

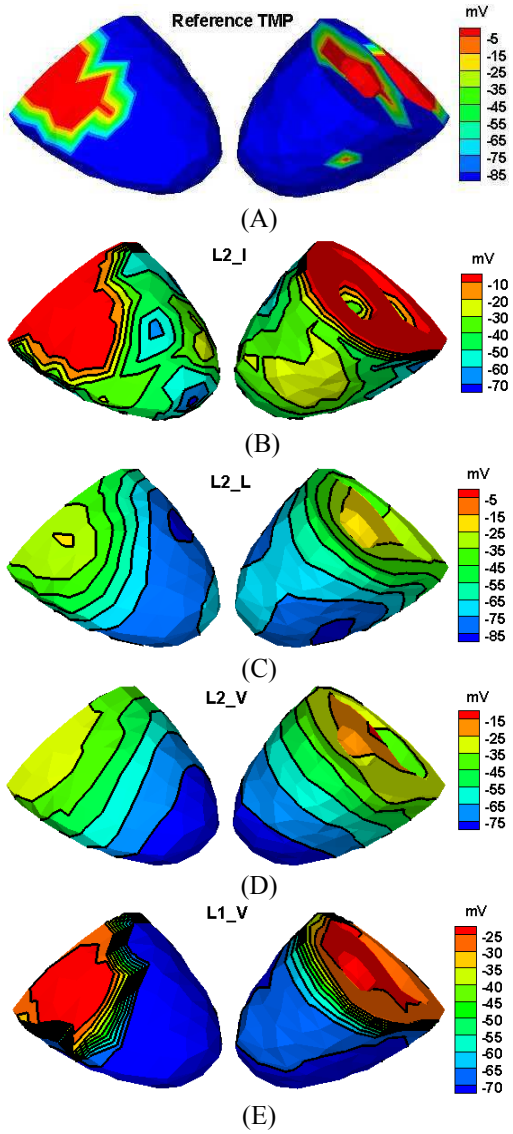


Figure 5. Reconstruction of TMP distribution on the ventricular surface from 30ms' BSP with SNR of 30dB with four methods: (A) reference TMP; (B) L2_I: CC is 0.64, RE is 0.70; (C) L2_L: CC is 0.76, RE is 0.46; (D) L2_V: CC is 0.69, RE is 0.67; (E) L1_V: CC is 0.74, RE is 0.47. The left column is the anterior view, while right column is the posterior view.

4. Discussion and conclusion

ECG inverse problem has been studied for several decades; lots of methods from mathematical or physical point of views have been proposed to tackle the ill-posedness of ECG inverse problem. However, it still hasn't been used in clinical application due to the gap between the inverse solutions and the clinical requirement. In this study, along with the development of L1 norm algorithms and the spatial sparse performance of TMP during the ventricular depolarization, a novel method L1_V was proposed to solve the TMP-based ECG inverse problem. Using a simulation study with realistic heart-torso model, the new method has been validated to provide more accurate cardiac TMP information.

In the future study, the proposed method needs to be tested by realistic measured BSPs and applied into some abnormal cardiac activities. Besides, the presented spatial constraint can be combined with additional temporal constraint as that in [5]to explore more accurate solutions.

Acknowledgements

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