Applying Lyapunov Exponents in Heart Rate Time Series to Identify the Anaerobic Threshold in Healthy Men

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Abstract

During the last years a lot of papers have appeared dealing with the applications to physiological time series of some parameters originally found in Dynamical Systems. In this work, we looked for the Largest Lyapunov Exponents (LLE) in heart rate time series of healthy men in order to verify if it was possible to find the Anaerobic Threshold (AT) in a non-invasive way using just the time series and the LLE extracted from them.

The work was undertaken in a group of 10 healthy individuals using a conventional electrocardiogram. The series of RR intervals, lasting for twelve minutes, were obtained in rest (supine and seated positions) and in a discontinuous protocol of dynamic exercise in seated position. The LLE were computed using the TISEAN system. The results were compared to those obtained by the measurement of AT using an autoregressive integrated moving-average model (ARIMA) and using the Kolmogorov-Sinai Entropy (KS). The Spearman test (rs) with $\alpha = 5\%$ applied to these three methods gave: a) for KS x ARIMA: $p < 0.01$, $rs = 0.93$; b) LLE x KS: $p < 0.01$, $rs = 0.94$ and c) LLE x ARIMA: $p < 0.01$, $rs = 0.93$. These results are very expressive concerning the use of LLE to find the AT.

1. Introduction

Physical exercise, as a spontaneous activity of daily life, involves complex physiological processes, which nowadays are not completely known. Most exercises result on movements of body segments and living organisms, related to different degrees of force generation to guarantee the survival of such organism in the environments. The muscle contraction involves the transformation of chemical energy stored as a molecule, adenosine tri-phosphate (ATP), into mechanical energy plus heat. In order to maintain this energy transformation for several seconds, or to be repeated for short intervals, the whole physiological system must be activated [1].

There are several kinds of muscle contraction, and therefore, different kinds of exercises. Dynamic exercise, also called isotonic or rhythmic exercise, is the most natural type of effort; it involves production of external work, and is an exercise type usually applied for evaluation of the aerobic capacity and cardio respiratory reserve. When a person undertakes exercise in a cycle-ergometer, with the power increasing gradually, the oxygen uptake (VO2) raises in linear way, until the point where additions of power levels do not modify the VO2; in these conditions, the VO2 reached its maximum value. The maximum VO2 is one of the most reliable parameters to measure the maximum magnitude of the oxygen transport involved in the chain of the physiological processes previously mentioned. In healthy individuals and patients with cardio respiratory diseases this point is rarely reached, because people interrupt the exercise before this limit, due to the discomfort manifested as fatigue or muscular pain. In these circumstances, it is possible to get only peak VO2, which always corresponding to a lower value compared to maximum VO2 [2].

Due to these facts, the peak VO2, generally, depends much more on the perception intensity (fatigue, muscle pain and air lack) of the effort magnitude, at a defined intensity of applied power, than on the saturation of the physiological systems. Therefore, it cannot consist in an objective measure of the transport of O2 in the dynamic exercise. Fortunately, another sub maximal parameter, the so called the anaerobic threshold (AT), has shown a reliable marker of aerobic capacity. Physiologically, this condition is the starting point of the rise of the blood lactic acid, which is related to the unbalance between its muscular production and its metabolic degradation by the liver and other tissues. There is a high level correlation between the maximum VO2 and the AT, which allows estimating maximum VO2 from the AT. From metabolic and neuro-humoral point of view, it is worth to state that the AT has a great physiological importance, being the limit between two different functional states. The study of the transport of O2 in healthy individuals and patients with cardio respiratory diseases had great advances in the last two decades, thanks to the possibility to measure the

AT from respiratory methods, using automatic and non-invasive procedures [1-3].

Some non-invasive methods and protocols have been used with the purpose of measuring the AT. The most commonly used methods nowadays that satisfy the required sensitivity and accuracy are based on the change in the pattern of respiratory and metabolic responses in AT. In such studies, it is mandatory to use equipment that allows the measurement of the mentioned variables at each breath, or the average value for each 15 seconds [1]. This equipment is expensive, which limits its use to few centers. Most recently, it has been shown that several physiological variables present a changing pattern coincident with the anaerobic threshold: this finding has allowed the use of heart rate response for quantification of this parameter [4].

The search for low cost, non-invasive methods for the identification of the AT, has raised interests from researchers that work in the signal processing applied to biological systems field [2-3,5-9]. But this is just a specific subject, because during the last years a lot of papers have appeared dealing with the applications to physiological time series of some parameters developed in Dynamical Systems [10-11].

We observed ten healthy men in dynamic exercises (DE) by various experimental protocols to see if the Largest Lyapunov Exponent (LLE) can be useful in detecting the AT.

2. Methodology

2.1. Subjects and materials

Ten healthy male volunteers have been studied: five young men (22 ± 1.5 years) and five middle-aged men (42 ± 2.5 years). They exhibited a sedentary life style. The DE were made with two experimental discontinuous protocols, that is, progressive and random power levels lasting fifteen minutes, with a rest period among them.

The DE was made with progressively growing resistance in a cycle ergometer (CORIVAL-400 model) with seated position (where the speed was around 60 rpm) and after 25 W we used increments of 10 and/or 5 W for 15 minutes. Between the different power levels there was a rest period in order to allow the cardiac frequency to return to its basal conditions.

If a slow slope in the cardiac frequency plot were identified in a given power, between the first and the twelfth minute, a last session with a 5 W smaller power would be performed. This procedure aimed to increase the power resolution where could possibly be found the AT.

The RR intervals have been measured, in milliseconds during DE.

2.2. Largest Lyapunov exponent

Lyapunov Exponents (EL) measure the rate of divergence or convergence of initially nearby trajectories in a phase space and estimate the amount of chaos in a system.

LE is often represented by the Greek letterλ. A system is allowed to develop from two slightly different initial states: x and x + ε. So after n iterations their divergence can be characterized as:

$$\varepsilon(n) = \varepsilon . e^{\lambda n} \quad (1)$$

where the exponent λ provides the average rate of divergence.

The Grassberger-Procaccia algorithm uses the probability to quantify information [12]. The Takens theorem [13] allows the construction of m-dimensional vectors ξ_i (where m is the embedding dimension) from only one time variable $\{x_i\}$, where $x_i = x(t_i)$ and i = 1, 2... N. In each vector ξ_i, $x(t_i)$ is its first coordinate, $x(t_{i+}\tau)$ is the second coordinate and $x(t_{i+(m-1)}.\tau)$ is the last coordinate (τ is the delay of the attractor reconstruction selected in this work as the value where the autocorrelation function takes e^{-1} where e is the Euler number) [14-16].

When you know the mathematical model that generated the dynamic system, the LLE can be calculated accurately from the algorithm proposed by Wolf [17]. But when we have only the experimental series then calculating the LLE is more complex and less accurate. Other authors also have already proposed algorithms: Sano and Sawada [18] and Eckmann et al. [19].

Kantz [20] and Rosenstein [21] developed, independently, methods which are substantively similar. They calculates the LLE by searching for all neighbors within a neighborhood of the reference trajectory and computes the average distance between neighbors and the reference trajectory as a function of time.

The Rosenstein algorithm for calculating the LLE from an experimental time series is accurate because it takes advantage of all the available data. It is fast, easy to implement, and robust to changes in the following quantities: embedding dimension, size of data set, reconstruction delay, and noise level.

We have calculated the LLE using the algorithm of Rosenstein from the TISEAN 3.0.1 (Time Series Analysis) free software package [22].

We ran the computer program done in C code by Rainer Hegger, modified in 2002 that is according to the algorithm Roseinstein and is part of the TISEAN with the following input: the original RR intervals series, dropping out the first and the final 1.5 minutes in order to assure for the studied sign a stability period; its corresponding delay (τ) of the Takens [13] was estimated from the autocorrelation function (Figure 1);

embedding dimension m = 10; window around the reference point which should be omitted t = 100 and minimal length scale for the neighborhood search r = 0.01.

The value of LLE was obtained by a linear regression of the curves as suggests Kantz [20] in the zone between 0 and 15.

Then, we have chosen the power of the cycle-ergometer corresponding to the smallest value of LLE as indicative of AT for each individual, because as LLE approaches zero, the attractor has no longer positive LE, that is, it is no longer strange attractor which shows a certain organization of the system, analogous to what we observed with respect to the KS entropy [6].

We have applied the one-sided t-test to compare the smallest value of LLE with others within a protocol.

To test the linear correlation between the indicative power values of LA found on this work with those found in [6] and [7] use Spearman rank correlation test. We examined a priori scatter diagrams to see if there were correlations between the methods.

Mann-Whitney test was used to compare groups: young and middle-aged. Progressive and random protocols were compared using the Wilcoxon test.

3. Objectives

Find out if LLE can be used as a mathematical tool to detect the AT in healthy individuals.

Compare the AT of young men with the middle-aged men.

Test the difference between the two protocols: progressive and random.

4. Results

We estimate the delay (τ) of each RR series using the autocorrelation functions (Figure 1).

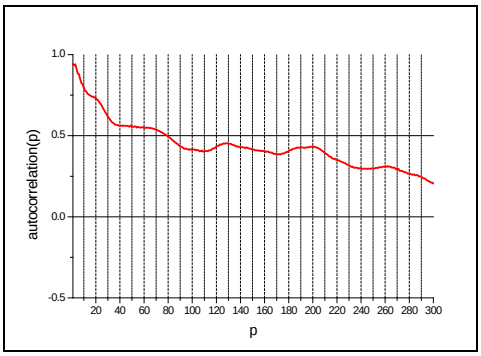

Figure 1. Autocorrelation function against the step (p) for an individual in DE; power: 115 W. Estimated τ = 80.

The intensity of power corresponding to the smallest value of LLE was taken as indicative of AT, for example: AT = 90 W in Table 1 (p-value=0.002).

Table 1. Relationship between power and LLE to a specific individual.

Power (W)	LLE
25	0.03801 ± 0.0479
35	0.02209 ± 0.00593
45	0.02682 ± 0.00604
55	0.04646 ± 0.00412
65	0.01938 ± 0.00321
75	0.01844 ± 0.005
85	0.01511 ± 0.00151
90	0.00955± 0.0018
95	0.01373 ± 0.00362
100	0.02031 ± 0.0015

Scatter diagrams showed a strong correlation between the methods (Figure 2).

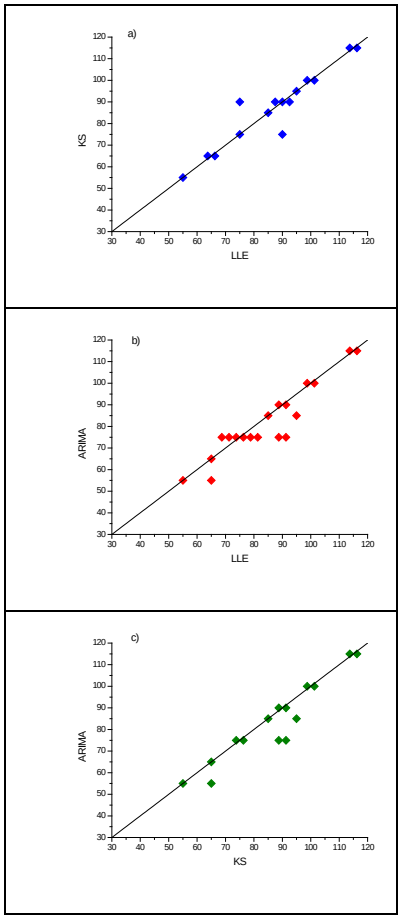

Figure 2. Scatter plots between the indicative powers

of the AT (Watts) for the procedures: a) LLE vs. KS; b) LLE vs. ARIMA and c) KS vs. ARIMA. The straight line in each scatter plot represents f(x) = x. The points side by side are coincident.

Table 2. Test Spearman Rank-order Correlation

	Sample size	rs[1]	p-value
LLE vs. KS	15	0.94	<0.01
LLE vs. ARIMA	19	0.93	<0.01
KS vs. ARIMA	15	0.93	<0.01

[1]Spearman rank-order correlation coefficient

There is no statistically significant difference for the young adult and AT middle age (p-value>0.05) even though the sample average of young people (94 W) was greater than that of middle-aged (79 W).

We also found no difference between the protocols (p-value>0.05) and in 67 per cent of cases the results were same.

5. Conclusions

We note that at the critical value of power, the LLE assumed a value significantly lower than the others. The largest Lyapunov exponent, then, approaches zero, at the AT, or is on the verge of changing signal. Since a chaotic dynamics has at least one positive Lyapunov exponent and the LE quantifies the sensitive dependence on initial conditions of the system, we conclude that the attractor reconstructed from our RR series presents a remarkable change in its dynamics at the critical power, that is, at the AT.

So, this is another non-invasive and low cost procedure to detect the AT. It is interesting to stress the strong association with the results obtained by the KS model and the ARIMA model.

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