A Framework for ECG Signal Preprocessing based on Quadratic Variation Reduction

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Abstract

ECG signals are corrupted by several kinds of noise and artifacts, which negatively affect any subsequent analysis. In the literature, the only approach that can handle any noise and artifacts corrupting the ECG is linear time-invariant filtering. However, it suffers from some important limitations regarding effectiveness and computational complexity. In this paper we propose a novel framework for ECG signal preprocessing based on the notion of quadratic variation reduction*. The framework is very general, since it can cope with all the different kinds of noise and artifacts that corrupt ECG records. It relies on a single algorithmic structure, thus enjoying an easy and robust implementation. Results show that the framework is effective in improving the quality of ECG, while preserving signal morphology. Moreover, it is very fast, even on long recordings, thus being perfectly suited for real-time applications and implementation on devices with reduced computational power, such as handheld devices.*

1. Introduction

The electrocardiogram (ECG) is the standard diagnostic tool for the routine assessment of heart function and the diagnosis of cardiac diseases. Unfortunately, the ECG signal is highly susceptible to several kinds of noise, such as electromyographic noise, power-line interference, motion artifacts, baseline wander, thermal and other measurement noise [1].

The most straightforward approach to ECG preprocessing is linear time-invariant (LTI) filtering, whether low-pass, high-pass or notch according to the kind of noise to remove [1]. This approach can be considered the only general method that is able to handle any noise and artifact corrupting the ECG. Indeed, it is implemented in every electrocardiograph used in the clinical practice [2]. However, it may introduce unacceptable distortions in the ECG [3] and residual noise might be present in the filtered signal. Moreover, although efficient implementations exist, sophisticated filters can hardly be implemented on devices with reduced computing power, such as mobile phones or handheld devices. Conversely, in recent years growing interest has been focused on the development of algorithms for assessing the quality of ECG running on mobile phones, which are nowadays pervasive [4]. This is particularly important when working in depressed areas of less-developed countries, where, due to a lack of adequate primary care capacity, ECG is acquired under not ideal conditions, often by untrained people, and with low-cost recorders, i.e., more sensitive to artifacts and noise [4].

To this end, in this paper we propose a novel framework for ECG signal preprocessing. It is based on the notion of *quadratic variation reduction* (QVR). The framework is very general, since it is able to cope with all different kinds of noise and artifacts that corrupt ECG recordings: baseline wander, narrowband artifacts (such as power-line interference), electromyographic and thermal noise. Moreover it can be used for single waves, e.g., P-waves and T-waves, and entire ECG records [5–8]. The main strengths of the framework are the following. First, it is *simple*, since a single algorithmic structure can easily handle all different kinds of noise by setting the appropriate values of its controlling parameters. Second, it is very *effective* in improving signal quality, while preserving ECG morphology. Finally, it is *fast* and computationally efficient, even on long recordings, thus being perfectly suitable for real-time implementation on handheld devices.

The paper is organized as follows. In Section 2 the notion of quadratic variation is introduced. The rationale behind the framework is described in Section 3. Sections 4 and 5 follow with simulation results and conclusions.

2. The quadratic variation

The effect of noise and artifacts on the ECG, *regardless* of their kind, consists in introducing additional "variability" into the observed signal with respect to the true one. Thus, provided that we introduce a suitable measure of variability, noise and artifacts can be suppressed by properly reducing the variability of the measured ECG.

To quantify the variability of a generic vector $\boldsymbol{x} = [x_1 \cdots x_n]^{\mathrm{T}} \in \mathbb{R}^n$, with $n \geq 2$, we introduce the *quadratic* *variation* (QV) of $\boldsymbol{x}$, denoted by $[\boldsymbol{x}]$, defined as

$$[\boldsymbol{x}] \doteq \sum_{k=1}^{n-1} (x_k - x_{k+1})^2 = \|\boldsymbol{D}\boldsymbol{x}\|^2 , \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean norm and $\boldsymbol{D}$ is the $(n-1) \times n$ matrix with entries

$$\boldsymbol{D}_{ij} = \delta_{i,j} - \delta_{i+1,j} \tag{2}$$

where $\delta_{i,j}$ is the Kronecker delta. The QV is a consistent measure of variability: for vectors affected by additive noise, regardless of the noise distribution, on average it is an increasing function of the noise variance [5].

The QV quantifies the variability of an entire ECG record or just a single wave, considered as a whole. However, the variability introduced by noise and artifacts is time-varying, since signal morphology changes over time, and noise and artifacts can be persistent, like thermal noise, or transient, like electromyographic noise. In this regard, the QV is also well suited to characterize the *local* variability of different segments of the ECG, like QRS complexes, P-waves or T-waves, which in general exhibit different local SNRs. Indeed, denoting by $\boldsymbol{x}(k_1, k_2) = [x_{k_1} \cdots x_{k_2}]^{\mathrm{T}}$, with $1 \le k_1 < k_2 \le n$, a generic subvector of $\boldsymbol{x}$, its quadratic variation can be expressed in terms of $\boldsymbol{x}$ as

$$[\boldsymbol{x}(k_1, k_2)] = \|\boldsymbol{D}\boldsymbol{S}(k_1, k_2)\boldsymbol{x}\|^2 = \|\boldsymbol{\mathfrak{D}}_i \boldsymbol{x}\|^2 , \tag{3}$$

where $\boldsymbol{\mathfrak{D}}_i = \boldsymbol{D}\boldsymbol{S}(k_i, k_{i+1})$ and $\boldsymbol{S}(k_1, k_2) \in \mathbb{R}^{n \times n}$ is a diagonal matrix defined as

$$S(k_1, k_2) = \mathrm{diag}\,(s_1, \cdots, s_n)$$

with

$$s_i = \begin{cases} 1\,, & k_1 \le i \le k_2 \\ 0\,, & \text{otherwise} \end{cases} .$$

Narrowband artifacts, such as power-line interference, require additional considerations. Their effect can be quantified using the QV in (1), however a different measure proved to be more effective in this case [7, 8], namely their energy content. To quantify it, let $\boldsymbol{X} = \boldsymbol{W}\boldsymbol{x}$ be the DFT [9] of $\boldsymbol{x}$, where $\boldsymbol{W}$ is the DFT matrix. Now, denote by $\tilde{\boldsymbol{W}}_f$ the matrix obtained stacking the rows of $\boldsymbol{W}$ corresponding to the harmonic components of the narrowband artifact centered around frequency f, which is to be rejected. The quadratic form

$$\mathcal{E}_{\boldsymbol{x}}(f) = \left\|\tilde{\boldsymbol{W}}_f \boldsymbol{x}\right\|^2 = \boldsymbol{x}^{\mathrm{T}} \mathrm{Re}\left\{\tilde{\boldsymbol{W}}_f^{\mathrm{H}} \tilde{\boldsymbol{W}}_f\right\} \boldsymbol{x} \tag{4}$$

quantifies the energy content of such an artifact, with $(\cdot)^{\mathrm{H}}$ denoting the transpose conjugate and $\mathrm{Re}\{\cdot\}$ the real part.

3. Denoising by QV reduction

In the following, $\boldsymbol{q} \in \mathbb{R}^n$ is the measured ECG, which is affected by noise and artifacts, and $\boldsymbol{x}$ the corresponding vector after denoising. The idea is that any kind of noise and artifacts affecting $\boldsymbol{q}$ can be removed by reducing the quadratic variation of the measured ECG, either locally or globally, and the energy content of narrowband artifacts. The magnitude of such a reduction, either local or global, is inversely related to the local or global SNR, respectively.

Let the vector $\boldsymbol{q}$ be decomposed into $L+1$ segments

$$\boldsymbol{q}(k_i, k_{i+1}) = [q_{k_i} \cdots q_{k_{i+1}}]^{\mathrm{T}}, \quad \text{for } i = 0, \ldots, L \tag{5}$$

with $0 \le L \le n-1$ and $1 = k_0 < k_1 < \cdots < k_L < k_{L+1} = n$. Segments in (5) denote distinct portions of the ECG, like QRS complexes, P-waves or T-waves, exhibiting different local SNRs. Note that two consecutive segments overlap in order to guarantee the absence of abrupt changes in the smoothed vector.

Denoising and artifact rejection can be achieved by solving the following convex optimization problem

$$\begin{cases} \underset{\boldsymbol{x} \in \mathbb{R}^n}{\text{minimize}} & \|\boldsymbol{x} - \boldsymbol{q}\|^2 \\ \\ \text{subject to} & [\boldsymbol{x}(k_i, k_{i+1})] \le a_i, \;\; i = 0, \ldots, L \\ & \mathcal{E}_{\boldsymbol{x}}(f_j) \le b_j, \qquad\;\; j = 1, \ldots, M \end{cases} \tag{6}$$

where $[\boldsymbol{x}(k_i, k_{i+1})]$ is the (local) quadratic variation of $\boldsymbol{x}(k_i, k_{i+1})$, and $\mathcal{E}_{\boldsymbol{x}}(f_j)$, as defined in (4), quantifies the energy content of the narrowband artifact centered around f_j. The constants $a_i > 0$, for $i = 0, \ldots, L$, control the degree of smoothness applied to each segment $\boldsymbol{q}(k_i, k_{i+1})$, and $b_i > 0$, for $i = 1, \ldots, M$, control the degree of rejection of the narrowband artifact centered around each f_i [8]. Note that we do *not* need to know in advance the appropriate values for a_i and b_j in any particular problem.

It is possible to prove that the solution of (6) is given by

$$\boldsymbol{x} = \left(\boldsymbol{I} + \sum_{i=0}^{L} \lambda_i \boldsymbol{\mathfrak{D}}_i^{\mathrm{T}} \boldsymbol{\mathfrak{D}}_i + \sum_{j=1}^{M} \nu_j \mathrm{Re}\left\{ \tilde{\boldsymbol{W}}_{f_j}^{\mathrm{H}} \tilde{\boldsymbol{W}}_{f_j} \right\} \right)^{-1} \boldsymbol{q} \tag{7}$$

where $\boldsymbol{I}$ denotes the identity matrix, and $\boldsymbol{\mathfrak{D}}_i$ and $\tilde{\boldsymbol{W}}_{f_j}$ are defined in (3) and (4), respectively. The parameters λ_i and ν_i in (7) are related to a_i and b_j in (6), but are used in their place to control the solution (7). Indeed, smoothing can be performed without caring about a_i and b_j in (6), by adapting λ_i and ν_j in (7) to meet some performance criteria, e.g., maximizing the SNR gain.

The problem (6) and its solution (7) have general validity, and can be adapted to suppress any particular kind of noise or artifact by setting the parameters in (7), namely L, M, λ_i, ν_j, and the decomposition (5). Noise and artifacts can be suppressed *one by one* or *jointly*. Moreover,

(7) can be also used to extract features or components from the ECG signal. To better appreciate the generality of (7), we report below some examples of its application.

• *Baseline wander removal*. Set $L = 0$ (only one segment $\boldsymbol{q}$, which is the entire ECG), $M = 0$ (no narrowband artifacts) and take λ_0 large enough (see [5]). The resulting $\boldsymbol{x}$ from (7) is the estimated baseline wander, which can be subtracted from $\boldsymbol{q}$. In [5, 6, 10] it is proven that this approach outperforms state-of-the-art algorithms in estimating baseline wander, while preserving the morphology of all waveforms in ECG, in particular the ST segment.

• *Power-line interference suppression*. Set $L = 0$ and $\lambda_0 = 0$ (only one segment $\boldsymbol{q}$, which can be the entire ECG or a single wave), $M > 0$ equal to the number of harmonics of the power-line interference to be suppressed, namely the fundamental at $50/60$ Hz and some higher harmonics. The resulting $\boldsymbol{x}$ from (7) has the power-line interference suppressed.

• *Smoothing single waves (e.g., P or T waves)*. Set $L = 0$ (only one segment $\boldsymbol{q}$, which is the single wave), $M = 0$ (no harmonic artifacts) and set λ_0 (see [11]). The resulting $\boldsymbol{x}$ from (7) is the smoothed wave. In [11] it is proven that this approach outperforms low-pass filtering. If narrowband artifacts are present, in particular $50/60$ Hz power-line noise, then $M > 0$. The special case $M = 1$ with harmonic artifacts is considered in [7].

• *Smoothing ECG records*. Set $L > 0$ (several segments corresponding to different portions of the ECG, i.e., QRS complexes, P-waves, T-waves, isoelectric segments, etc., with $\boldsymbol{q}$ being the ECG record), $M = 0$ (no harmonic artifacts) and set λ_i (see [12]). Note that since the ECG is pseudo-periodic the number of independent λ_i is strongly reduced [12]. The resulting $\boldsymbol{x}$ from (7) is the smoothed ECG. In [12] it is proven that this approach largely outperforms low-pass filtering. If narrowband artifacts are present then $M > 0$. The special case $M = 1$ is considered in [8].

Finally, even though parameters λ_i and ν_j interact, λ_i mainly controls the smoothness of the ith segment $\boldsymbol{x}(k_i, k_{i+1})$, whereas ν_j mainly controls the degree of rejection of the narrowband artifact centered around f_j.

3.1. Computational issues

Formula (7) involves matrix inversion, which has complexity $O(n^3)$. However, it is possible to prove that due to the special structure of the matrices involved, formula (7) can be computed with complexity ranging from $O(n)$ to $O(n \log n)$. In particular, when $M = 0$, i.e., there are no narrowband artifacts, (7) can be computed by direct methods with complexity $O(n)$, *regardless* of L. In the case $M > 0$ and $L = 0$, i.e., only narrowband artifacts and measurement noise are present, (7) can be computed by direct methods with complexity $O(n \log n)$. Whereas in the general case $L > 0$ and $M > 0$, (7) can be evaluated with complexity $O(n \log n)$ using the conjugate gradient method.

4. Simulation results

Performance of the proposed framework was extensively investigated in several papers [5–8], considering all kinds of noise and artifacts that usually corrupt the ECG.

Here we report an example of the performance of the framework on an ECG record corrupted by baseline wander, narrowband artifacts, measurement noise and muscle artifacts. We considered the ECG record mitdb/118 from the MIT-BIH Arrhythmia Database [13] from PhysioNet [14]. It is a two-channel recording acquired at a sampling frequency of 360 Hz with 11–bit resolution and is slightly affected by baseline wander and measurement noise. Such a record, denoted in the following by $\boldsymbol{q}_0$, was further corrupted with baseline wander, namely $\boldsymbol{b}$, and electromyographic artifact, namely $\boldsymbol{m}$, from the records nstdb/bw and nstdb/ma, respectively, from the MIT-BIH Noise Stress Test Database [15] from PhysioNet [14]. These are both two-channel recordings acquired at a sampling frequency of 360 Hz from a physically active volunteer placing the electrodes on the limbs in positions in which the subject's ECG was not visible [15]. As regards narrowband artifacts, we considered sine waves at 30 Hz and 50 Hz accounting for a generic in-band artifact and power-line interference, respectively. Such waves have time-varying amplitudes obtained by low-pass filtering with cut-off frequency 2 Hz two independent realizations of zero-mean white Gaussian noise. The corrupted signal, denoted by $\boldsymbol{q} = \boldsymbol{q}_0 + \boldsymbol{b} + \boldsymbol{m} + \boldsymbol{d}$, with $\boldsymbol{d}$ the sum of the two sine waves, is reported in Fig. 1 (red) together with the original record $\boldsymbol{q}_0$ (blue).

In Fig. 2 we report the original ECG $\boldsymbol{q}_0$ (blue) and the smoothed record $\boldsymbol{x}$ (red) resulting from noise and artifacts suppression using the proposed framework. Smoothing was achieved by firstly estimating and removing baseline wander and, then, jointly reducing the other kinds of noise and artifacts. As regards baseline wander estimation ($L = 0$, $M = 0$), we roughly set $\lambda = 3600$, following [5]. Joint noise and harmonic artifact suppression was applied with different smoothing parameters to the following segments: isoelectric PQ, ST and TP segments (λ_{iso}), P-waves (λ_{P}), QRS complexes (λ_{QRS}), and T-waves (λ_{T}). The corresponding smoothing parameters were roughly set to $\lambda_{\text{iso}} = 3$, $\lambda_{\text{P}} = \lambda_{\text{T}} = 1$, and $\lambda_{\text{QRS}} = 0$ (i.e., no smoothing for QRS complexes). Concerning harmonic artifacts, we roughly set $\nu_1 = \nu_2 = 40$. Fig. 2 highlights that the proposed framework can cope with all the different kinds of noise and artifacts that usually corrupt ECG records, even without optimization of the smoothing parameters. Note that the mismatch between $\boldsymbol{q}_0$ and $\boldsymbol{x}$ in Fig. 2 is due to the

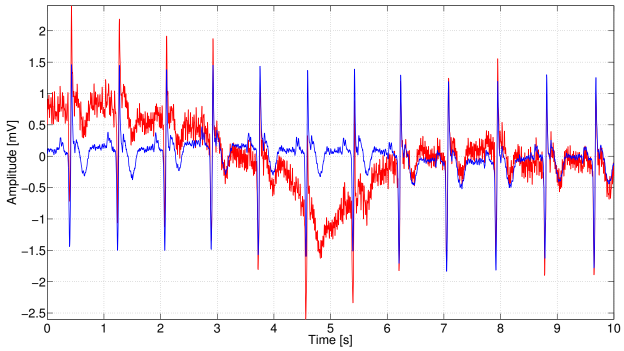

Figure 1. Real ECG ($\boldsymbol{q}_0$, blue) corrupted with additional baseline wander, muscle and harmonic artifacts ($\boldsymbol{q}$, red).

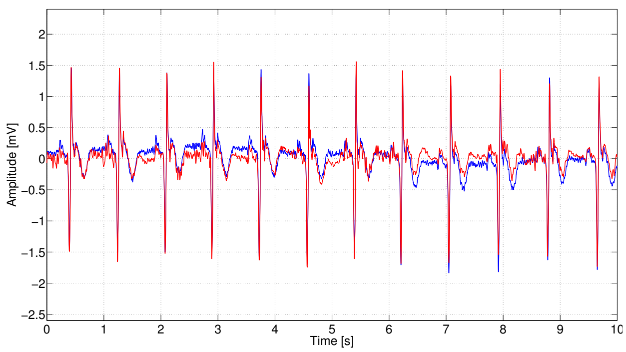

Figure 2. ECG record from Fig. 1 ($\boldsymbol{q}_0$, blue) and reconstructed version using the proposed framework ($\boldsymbol{x}$, red).

fact that the original record $\boldsymbol{q}_0$ was not noise free, but exhibited itself some baseline drift and measurement noise, which have been correctly removed using the framework.

5. Conclusions

In this paper we present a novel framework for ECG signal preprocessing, which is based on the notion of *quadratic variation reduction*. The quadratic variation is introduced as a measure of variability for sampled signals. The proposed framework is very general and can cope with all kinds of noise and artifacts that corrupt the ECG. In particular, it can effectively handle both transient and persistent noise, and can be applied to entire ECG records as well as to single waves, like P-waves or T-waves. The framework relies on a single algorithmic structure that can be easily adapted to cope with different kinds of noise and artifacts. Results show that it is effective in improving signal quality, while preserving ECG morphology. Moreover, it is very fast, even on long recordings, thus being perfectly suitable for real-time applications and implementation on devices with reduced computational power, such as handheld devices.

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